Organisation und Architektur von Rechnern

Lecture 03

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http://www.informatik.uni-kiel.de/rtsys/teaching/v-sysinf2

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Last Time: Bits & Bytes

- Bits, Bytes, Words
- Decimal, binary, hexadecimal representation
- Virtual memory space, addressing, byte ordering
- Boolean algebra
- Bit versus logical operations in C

Today: Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating
- Addition, negation, multiplication, shifting
- Summary

Integer C Puzzles

- Taken from old exams
- Assume 32-bit word size, two's complement integers
- For each of the following C expressions, either:
  - Argue that is true for all argument values
  - Give example where not true

Initialization

```
int x = foo();

int y = bar();

unsigned ux = x;

unsigned uy = y;
```

❑ **x < 0**	⇒	**((x*2) < 0)**
❑ **ux >= 0**		
❑ **x & 7 == 7**	⇒	**(x<<30) < 0**
❑ **ux > -1**		
❑ **x > y**	⇒	**-x < -y**
❑ **x * x >= 0**		
❑ **x > 0 && y > 0**	⇒	**x + y > 0**
❑ **x >= 0**	⇒	**-x <= 0**
❑ **x <= 0**	⇒	**-x >= 0**

Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- **C `short` 2 bytes long**

	Decimal	Hex	Binary
`x`	15213	`3B 6D`	`00111011 01101101`
`y`	-15213	`C4 93`	`11000100 10010011`

- **Sign Bit**
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative
    - 1 for negative

Encoding Example (Cont.)

```
x =      15213: 00111011 01101101
y =     -15213: 11000100 10010011
```

Weight	15213		-15213	
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
Sum		**15213**		**-15213**

Numeric Ranges

- **Unsigned Values**
  - *UMin* = 0

    000…0
  - *UMax* = $2^w - 1$

    111…1

- **Two's Complement Values**
  - *TMin* = -2^{w-1}

    100…0
  - *TMax* = $2^{w-1} - 1$

    011…1

- **Other Values**
  - Minus 1

    111…1

Values for *W* = 16

	Decimal	Hex	Binary
`UMax`	65535	FF FF	11111111 11111111
`TMax`	32767	7F FF	01111111 11111111
`TMin`	-32768	80 00	10000000 00000000
`-1`	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Values for Different Word Sizes

	W			
	8	**16**	**32**	**64**
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

- **Observations**
  - $|TMin| = TMax + 1$
    - Asymmetric range
  - $UMax = 2 * TMax + 1$

- **C Programming**
  - #include <limits.h>
  - Declares constants, e.g.,
    - ULONG_MAX
    - LONG_MAX
    - LONG_MIN
  - Values platform specific

Unsigned & Signed Numeric Values

X	B2U(*X*)	B2T(*X*)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

- **Equivalence**
  - Same encodings for nonnegative values
- **Uniqueness**
  - Every bit pattern represents unique integer value
  - Each representable integer has unique bit encoding
- **⇒ Can Invert Mappings**
  - U2B(x) = B2U^{-1}(x)
    - Bit pattern for unsigned integer
  - T2B(x) = B2T^{-1}(x)
    - Bit pattern for two's comp integer

Today: Integers

- Representation: unsigned and signed
- **Conversion, casting**
- Expanding, truncating
- Addition, negation, multiplication, shifting
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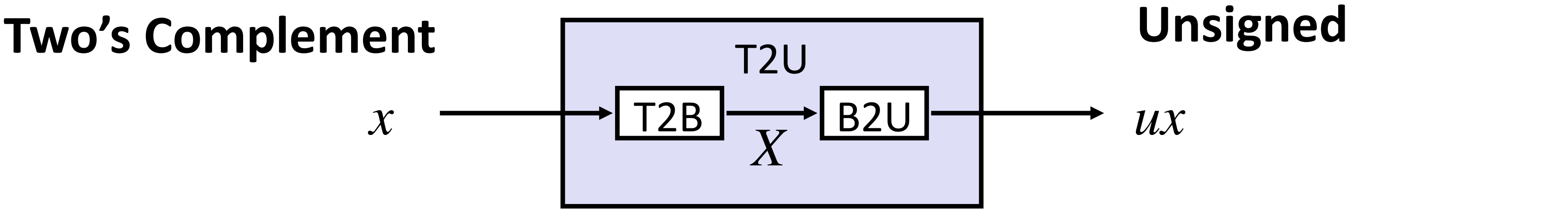
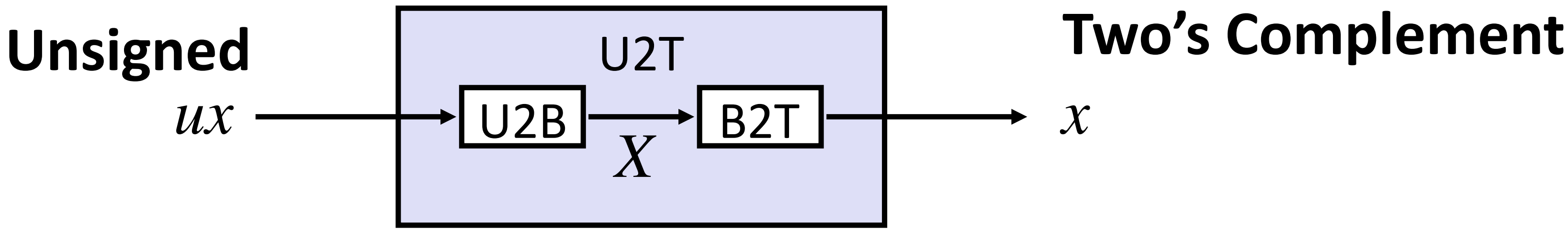
Mapping Between Signed & Unsigned



Maintain Same Bit Pattern



Maintain Same Bit Pattern

- **Mappings been unsigned and two's complement numbers:**
  keep bit representations and reinterpret

Mapping Signed ↔ Unsigned

Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

$\longrightarrow \boxed{\text{T2U}} \longrightarrow$

$\longleftarrow \boxed{\text{U2T}} \longleftarrow$

Mapping Signed ↔ Unsigned

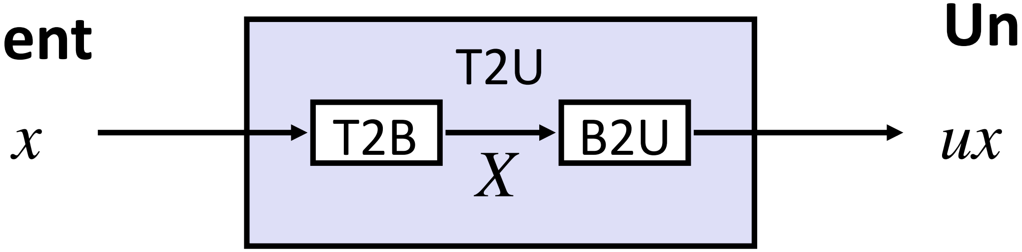
Bits	Signed	Unsigned
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	-8	8
1001	-7	9
1010	-6	10
1011	-5	11
1100	-4	12
1101	-3	13
1110	-2	14
1111	-1	15

=

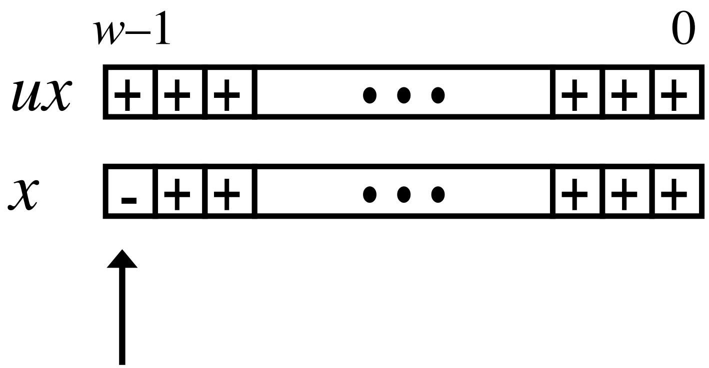
+16

Relation between Signed & Unsigned

Two's Complement **Unsigned**



Maintain Same Bit Pattern

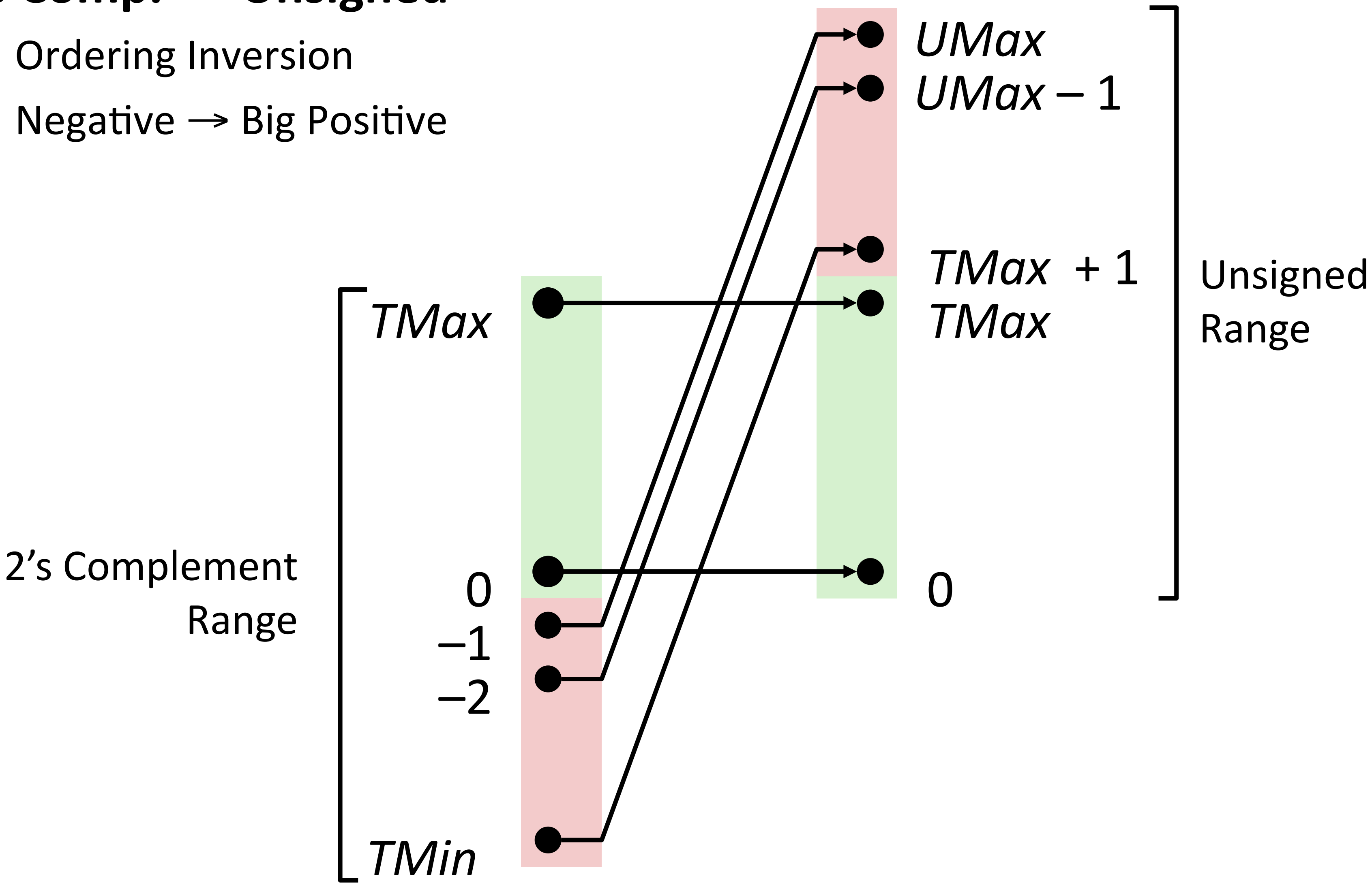


Large negative weight
becomes
Large positive weight

$$ux = \begin{cases} x & x \ge 0 \\ x + 2^w & x < 0 \end{cases}$$

Conversion Visualized

- **2's Comp. → Unsigned**
  - Ordering Inversion
  - Negative → Big Positive



Signed vs. Unsigned in C

- **Constants**
  - By default are considered to be ***signed integers***
  - Unsigned if have “U” as suffix

    ```
    0U, 4294967259U
    ```

- **Casting**
  - Explicit casting between signed & unsigned same as U2T and T2U

    ```
    int tx, ty;
    unsigned ux, uy;
    tx = (int) ux;
    uy = (unsigned) ty;
    ```

  - Implicit casting also occurs via assignments and procedure calls

    ```
    tx = ux;
    uy = ty;
    ```

Casting Surprises

- **Expression Evaluation**
  - If mix unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for $W = 32$: **TMIN = -2,147,483,648 , TMAX = 2,147,483,647**

Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U	**==**	**unsigned**
-1	0	**<**	**signed**
-1	0U	**>**	**unsigned**
2147483647	-2147483647-1	**>**	**signed**
2147483647U	-2147483647-1	**<**	**unsigned**
-1	-2	**>**	**signed**
(unsigned)-1	-2	**>**	**unsigned**
2147483647	2147483648U	**<**	**unsigned**
2147483647	(int) 2147483648U	**>**	**signed**

Code Security Example

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

- **Similar to code found in FreeBSD's implementation of getpeername**
- **There are legions of smart people trying to find vulnerabilities in programs**

Typical Usage

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

```
#define MSIZE 528

void getstuff() {
    char mybuf[MSIZE];
    copy_from_kernel(mybuf, MSIZE);
    printf("%s\n", mybuf);
}
```

Malicious Usage

```
/* Declaration of library function memcpy */
void *memcpy(void *dest, void *src, size_t n);
```

```
/* Kernel memory region holding user-accessible data */
#define KSIZE 1024
char kbuf[KSIZE];

/* Copy at most maxlen bytes from kernel region to user buffer */
int copy_from_kernel(void *user_dest, int maxlen) {
    /* Byte count len is minimum of buffer size and maxlen */
    int len = KSIZE < maxlen ? KSIZE : maxlen;
    memcpy(user_dest, kbuf, len);
    return len;
}
```

```
#define MSIZE 528

void getstuff() {
    char mybuf[MSIZE];
    copy_from_kernel(mybuf, -MSIZE);
    . . .
}
```

Summary
Casting Signed ↔ Unsigned: Basic Rules

- **Bit pattern is maintained**
- **But reinterpreted**
- **Can have unexpected effects: adding or subtracting 2^w**

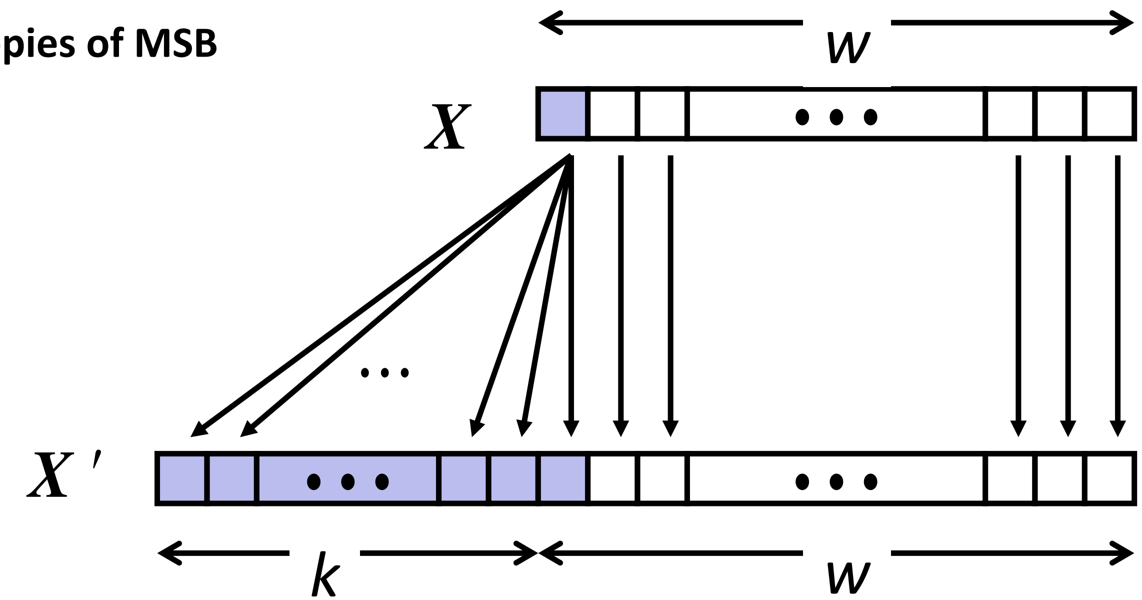
- **Expression containing signed and unsigned int**
  - int is cast to unsigned!!

Today: Integers

- Representation: unsigned and signed
- Conversion, casting
- **Expanding, truncating**
- Addition, negation, multiplication, shifting
- Summary

Sign Extension

- **Task:**
  - Given *w*-bit signed integer *x*
  - Convert it to *w*+*k*-bit integer with same value
- **Rule:**
  - Make *k* copies of sign bit:
  - $X' = \underbrace{x_{w-1},\ldots, x_{w-1}}_{k \text{ copies of MSB}}, x_{w-1}, x_{w-2},\ldots, x_0$



Sign Extension Example

```
short int x =  15213;
int       ix = (int) x;
short int y = -15213;
int       iy = (int) y;
```

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101
y	-15213	C4 93	11000100 10010011
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011

- Converting from smaller to larger integer data type
- C automatically performs sign extension

Summary:
Expanding, Truncating: Basic Rules

- **Expanding (e.g., short int to int)**
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result

- **Truncating (e.g., unsigned to unsigned short)**
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small numbers yields expected behaviour

Today: Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating
- **Addition, negation, multiplication, shifting**
- Summary

Negation: Complement & Increment

- **Claim: Following Holds for 2's Complement**

  `~x + 1 == -x`

- **Complement**
  - Observation: **`~x + x == 1111…111 == -1`**

	x	1	0	0	1	1	1	0	1
+	~x	0	1	1	0	0	0	1	0
	-1	1	1	1	1	1	1	1	1

- **Complete Proof?**

Complement & Increment Examples

x = 15213

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
~x	-15214	C4 92	11000100 10010010
~x+1	-15213	C4 93	11000100 10010011
y	-15213	C4 93	11000100 10010011

x = 0

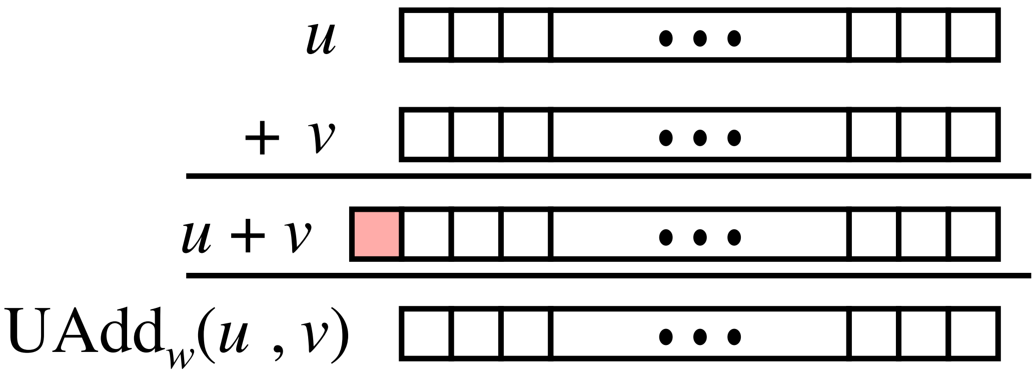
	Decimal	Hex	Binary
0	0	00 00	00000000 00000000
~0	-1	FF FF	11111111 11111111
~0+1	0	00 00	00000000 00000000

Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- **Standard Addition Function**
  - Ignores carry output
- **Implements Modular Arithmetic**

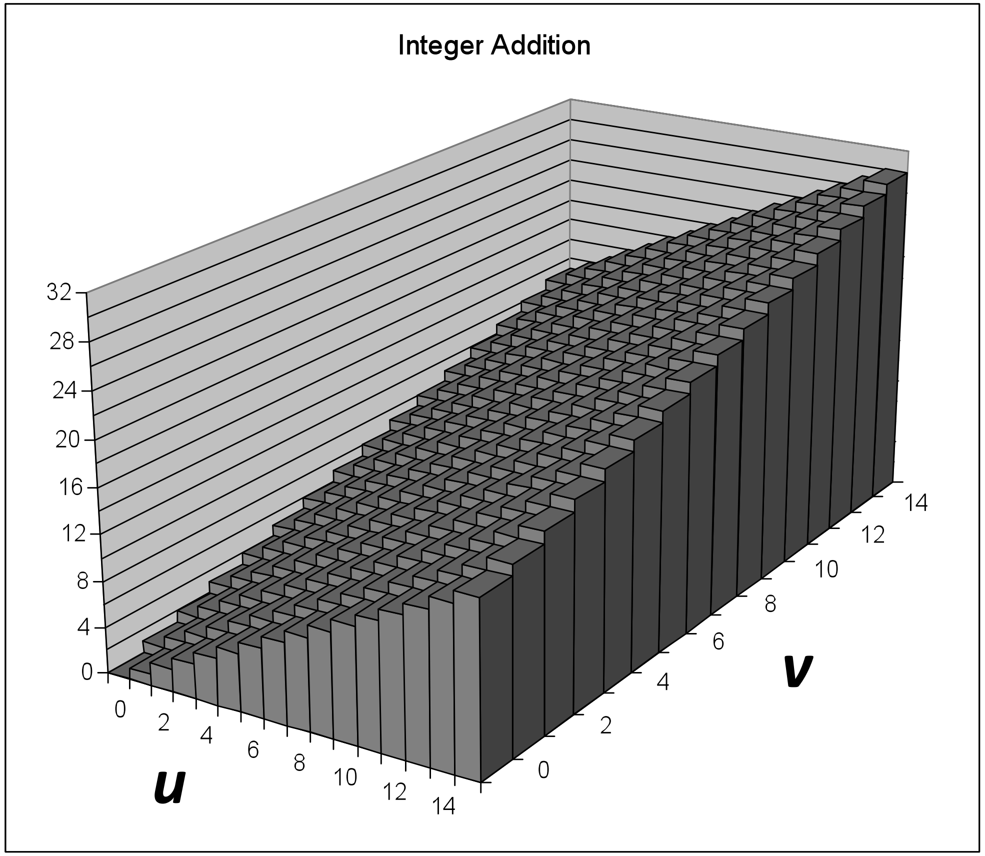
  $s \quad = \quad \mathrm{UAdd}_w(u\ ,\ v) \quad = \quad (u + v) \bmod 2^w$

$$UAdd_w(u,v) \quad = \quad \begin{cases} u+v & u+v < 2^w \\ u+v-2^w & u+v \geq 2^w \end{cases}$$

Visualizing (Mathematical) Integer Addition

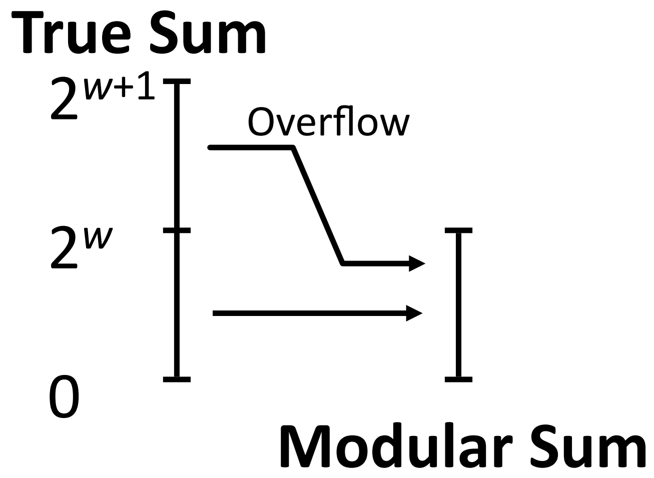
- **Integer Addition**
  - 4-bit integers *u*, *v*
  - Compute true sum $Add_4(u, v)$
  - Values increase linearly with *u* and *v*
  - Forms planar surface

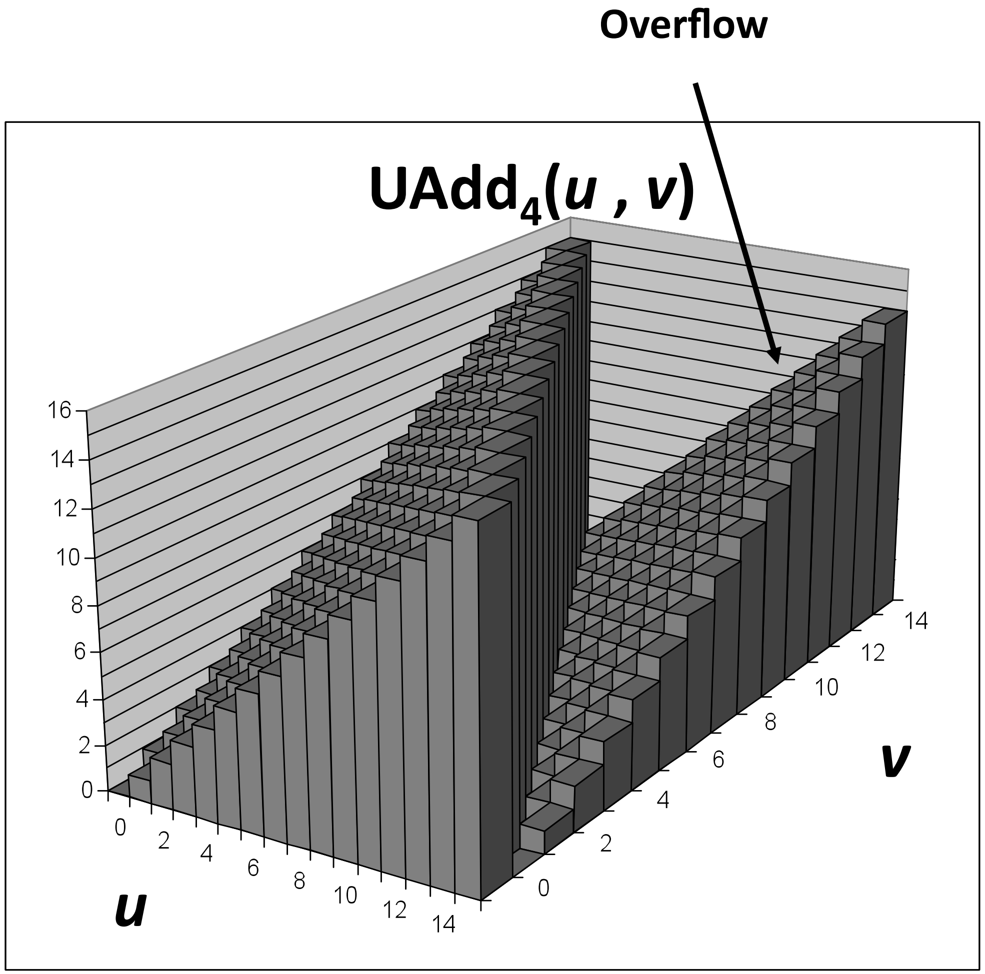
$Add_4(u, v)$



Visualizing Unsigned Addition

- **Wraps Around**
  - If true sum ≥ 2^w
  - At most once





Mathematical Properties

- **Modular Addition Forms an *Abelian Group***
  - **Closed** under addition

    $0 \le \mathrm{UAdd}_w(u\ , v) \le 2^w - 1$
  - **Commutative**

    $\mathrm{UAdd}_w(u\ , v) = \mathrm{UAdd}_w(v\ , u)$
  - **Associative**

    $\mathrm{UAdd}_w(t, \mathrm{UAdd}_w(u\ , v)) = \mathrm{UAdd}_w(\mathrm{UAdd}_w(t, u\ ), v)$
  - **0** is additive identity

    $\mathrm{UAdd}_w(u\ , 0) = u$
  - Every element has additive **inverse**
    - Let $\mathrm{UComp}_w(u\ ) = (2^w - u) \bmod 2^w$

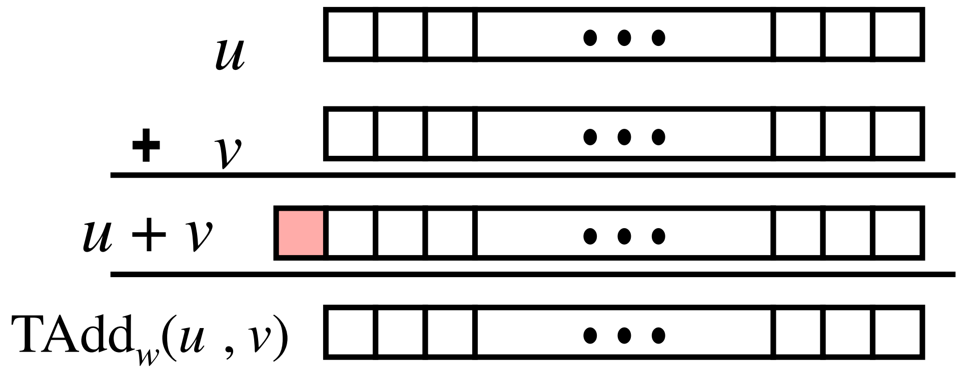
      $\mathrm{UAdd}_w(u\ , \mathrm{UComp}_w(u\ )) = 0$

Two's Complement Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



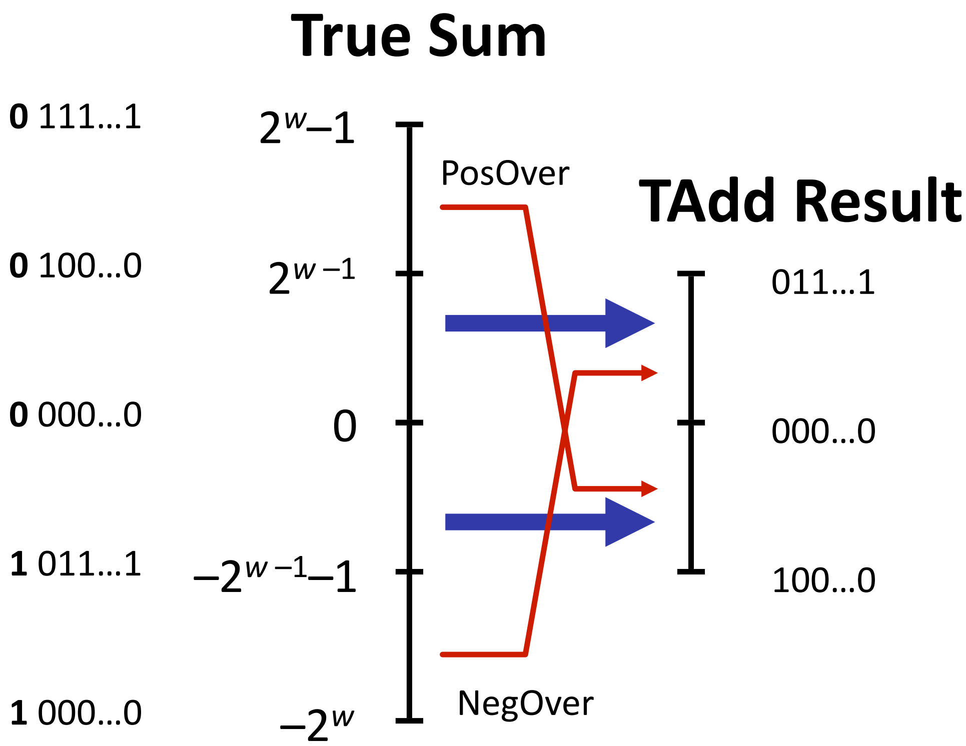
- **TAdd and UAdd have Identical Bit-Level Behavior**
  - Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

  - Will give `s == t`

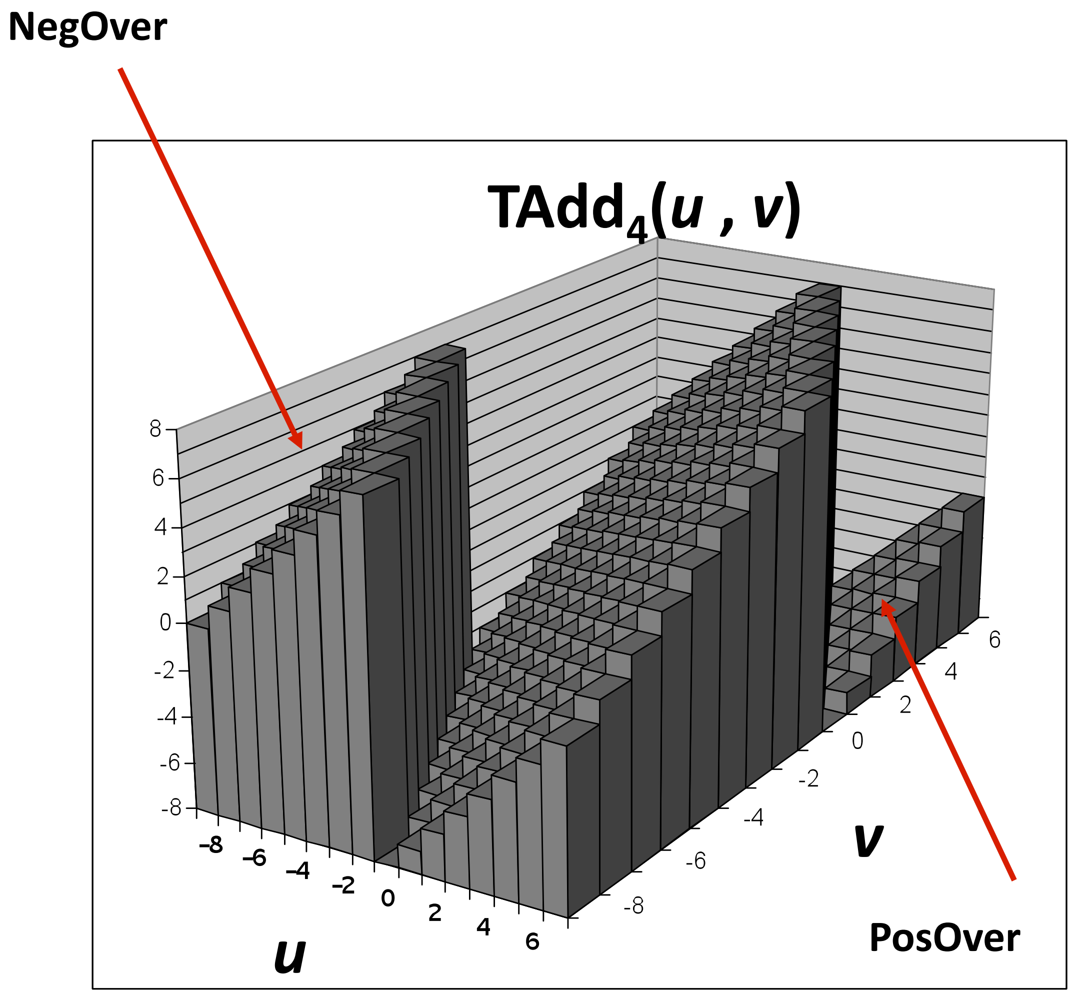
TAdd Overflow

- **Functionality**
  - True sum requires *w*+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



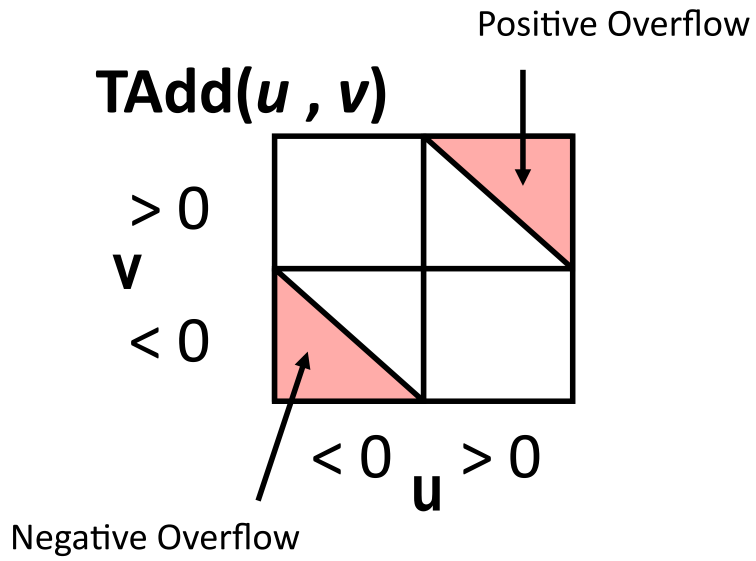
Visualizing 2's Complement Addition

- **Values**
  - 4-bit two's comp.
  - Range from -8 to +7
- **Wraps Around**
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once



Characterizing TAdd

- **Functionality**
  - True sum requires *w*+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



$$TAdd_w(u,v) = \begin{cases} u+v+2^w & u+v < TMin_w \quad \text{(NegOver)} \\ u+v & TMin_w \le u+v \le TMax_w \\ u+v-2^w & TMax_w < u+v \quad \text{(PosOver)} \end{cases}$$

Mathematical Properties of TAdd

- **Isomorphic Group to unsigneds with UAdd**
  - $\text{TAdd}_w(u, v) = \text{U2T}(\text{UAdd}_w(\text{T2U}(u), \text{T2U}(v)))$
    - Since both have identical bit patterns

- **Two's Complement Under TAdd Also Forms an Abelian Group**
  - Closed, Commutative, Associative, 0 is additive identity
  - Every element has additive inverse

$$TComp_w(u) = \begin{cases} -u & u \neq TMin_w \\ TMin_w & u = TMin_w \end{cases}$$

Multiplication

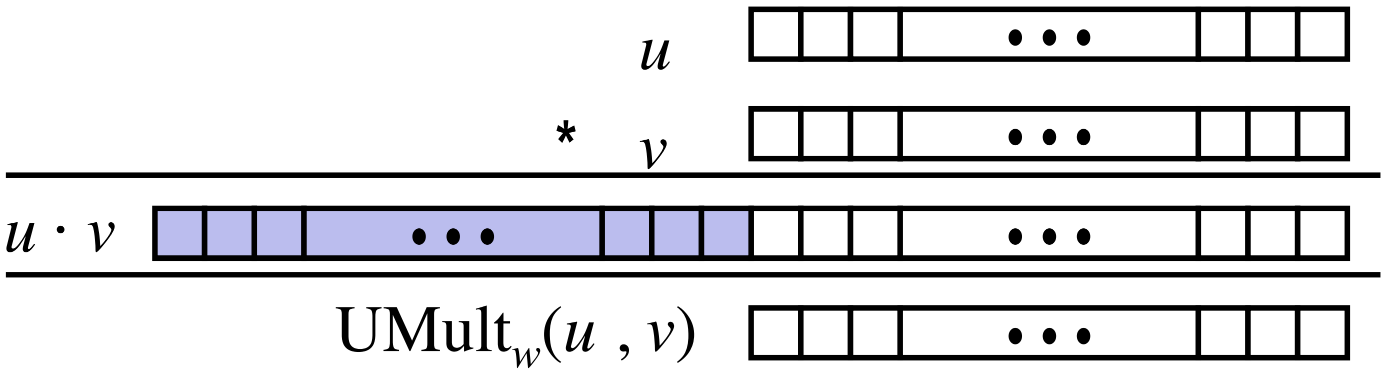
- **Computing Exact Product of w-bit numbers x, y**
  - Either signed or unsigned
- **Ranges**
  - Unsigned: $0 \le x * y \le (2^w - 1)^2 = 2^{2w} - 2^{w+1} + 1$
    - Up to $2w$ bits
  - Two's complement min: $x * y \ge (-2^{w-1})*(2^{w-1}-1) = -2^{2w-2} + 2^{w-1}$
    - Up to $2w-1$ bits
  - Two's complement max: $x * y \le (-2^{w-1})^2 = 2^{2w-2}$
    - Up to $2w$ bits, but only for $(TMin_w)^2$
- **Maintaining Exact Results**
  - Would need to keep expanding word size with each product computed
  - Done in software by "arbitrary precision" arithmetic packages

Unsigned Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits



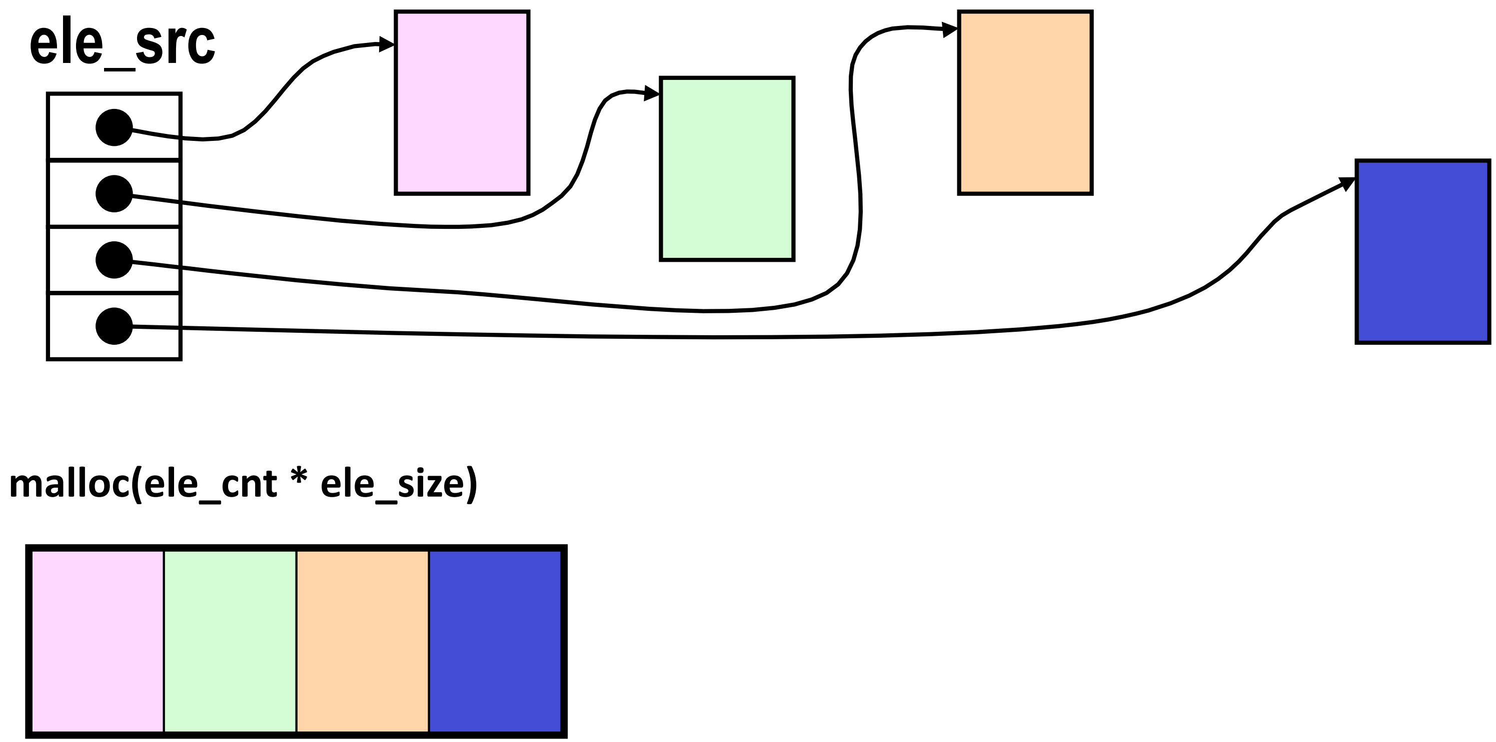
- **Standard Multiplication Function**
  - Ignores high order *w* bits
- **Implements Modular Arithmetic**

  $\mathrm{UMult}_w(u\,,\,v) = \quad (u \cdot v) \mod 2^w$

Code Security Example #2

- **SUN XDR library**
  - Widely used library for transferring data between machines

```
void* copy_elements(void *ele_src[], int ele_cnt, size_t ele_size);
```



XDR Code

```
void* copy_elements(void *ele_src[], int ele_cnt, size_t ele_size) {
    /*
     * Allocate buffer for ele_cnt objects, each of ele_size bytes
     * and copy from locations designated by ele_src
     */
    void *result = malloc(ele_cnt * ele_size);
    if (result == NULL)
        /* malloc failed */
        return NULL;
    void *next = result;
    int i;
    for (i = 0; i < ele_cnt; i++) {
        /* Copy object i to destination */
        memcpy(next, ele_src[i], ele_size);
        /* Move pointer to next memory region */
        next += ele_size;
    }
    return result;
}
```

XDR Vulnerability

malloc(ele_cnt * ele_size)

- **What if:**
  - `ele_cnt` $= 2^{20} + 1$
  - `ele_size` $= 4096$ $= 2^{12}$
  - Allocation = ??

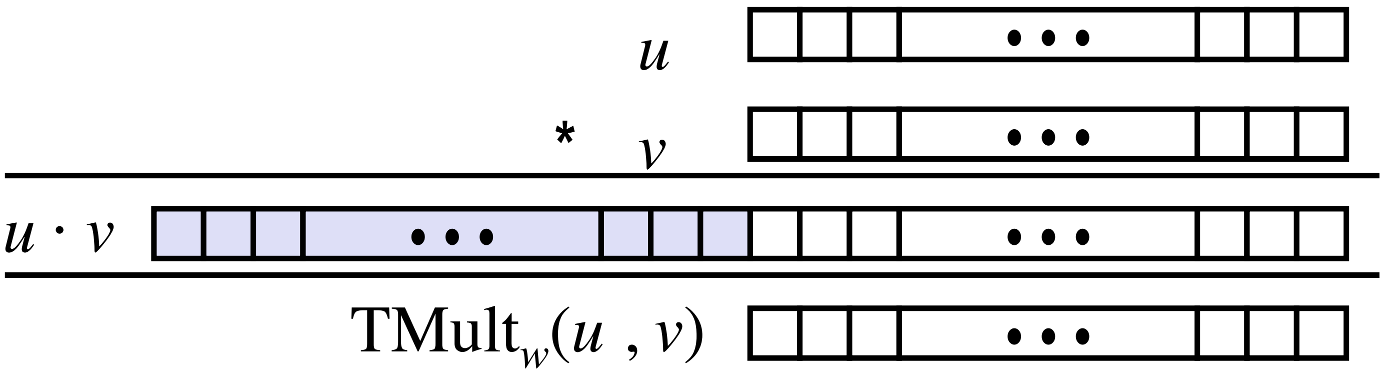
- **How can I make this function secure?**

Signed Multiplication in C

Operands: *w* bits

True Product: 2**w* bits

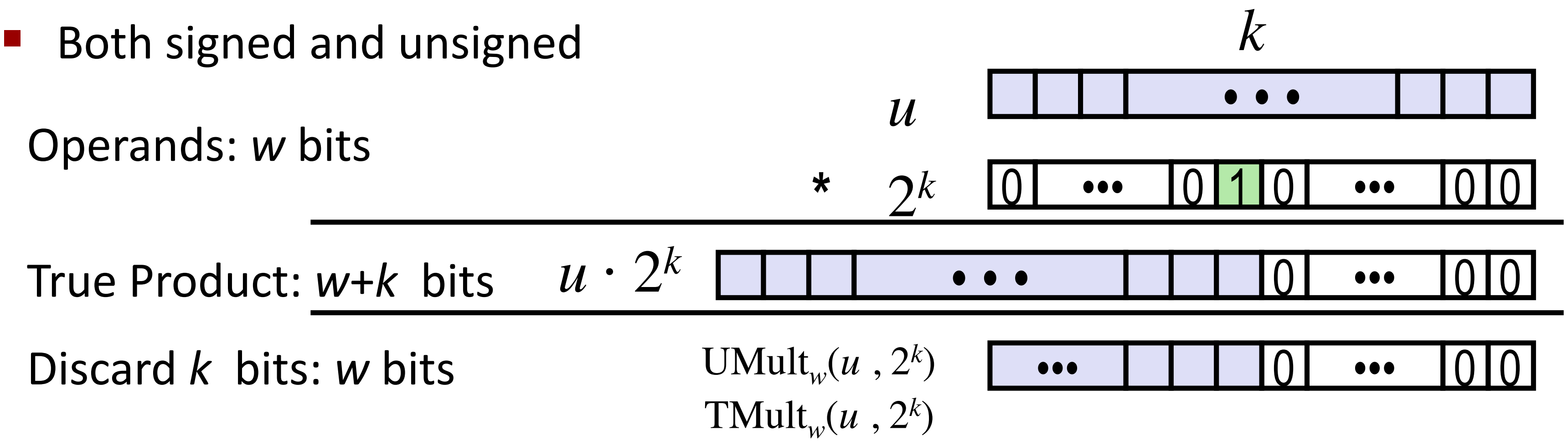
Discard *w* bits: *w* bits



- **Standard Multiplication Function**
  - Ignores high order *w* bits
  - Some of which are different for signed vs. unsigned multiplication
  - Lower bits are the same

Power-of-2 Multiply with Shift

- **Operation**
  - `u << k` gives `u * 2`^{k}
  - Both signed and unsigned



- **Examples**
  - `u << 3 == u * 8`
  - `u << 5 - u << 3 == u * 24`
  - Most machines shift and add faster than multiply
    - Compiler generates this code automatically

Compiled Multiplication Code

C Function

```
int mul12(int x)
{
  return x*12;
}
```

Compiled Arithmetic Operations

```
  leal (%eax,%eax,2), %eax
  sall $2, %eax
```

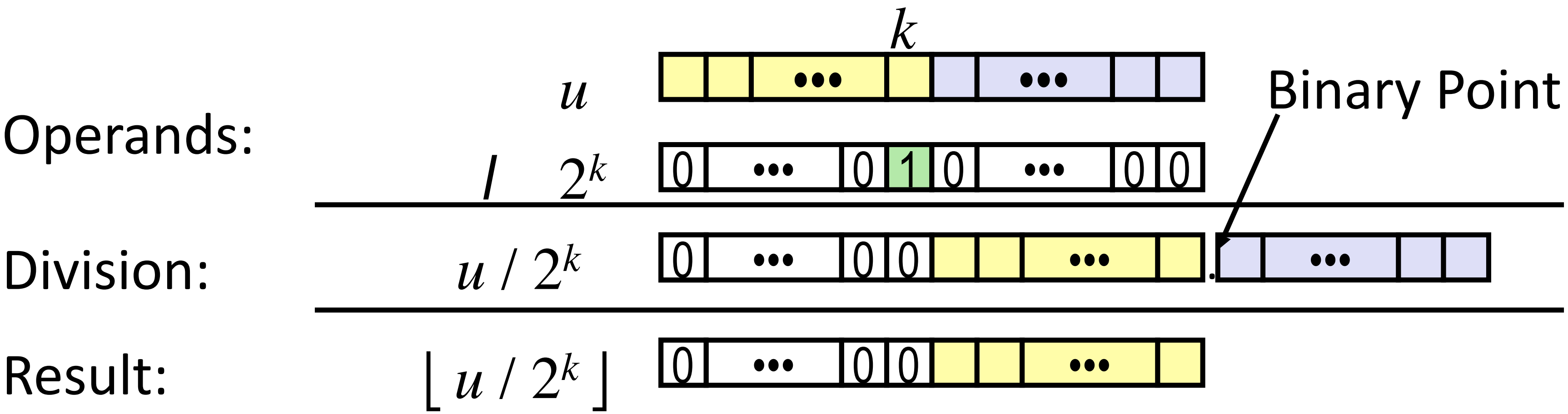
Explanation

```
  t <- x+x*2
  return t << 2;
```

- C compiler automatically generates shift/add code when multiplying by constant

Unsigned Power-of-2 Divide with Shift

- **Quotient of Unsigned by Power of 2**
  - `u >> k` gives $\lfloor$ `u / 2`^{k} $\rfloor$
  - Uses logical shift



	Division	Computed	Hex	Binary
`x`	15213	15213	`3B 6D`	`00111011 01101101`
`x >> 1`	7606.5	7606	`1D B6`	`00011101 10110110`
`x >> 4`	950.8125	950	`03 B6`	`00000011 10110110`
`x >> 8`	59.4257813	59	`00 3B`	`00000000 00111011`

Compiled Unsigned Division Code

C Function

```
unsigned udiv8(unsigned x)
{
  return x/8;
}
```

Compiled Arithmetic Operations

```
  shrl $3, %eax
```

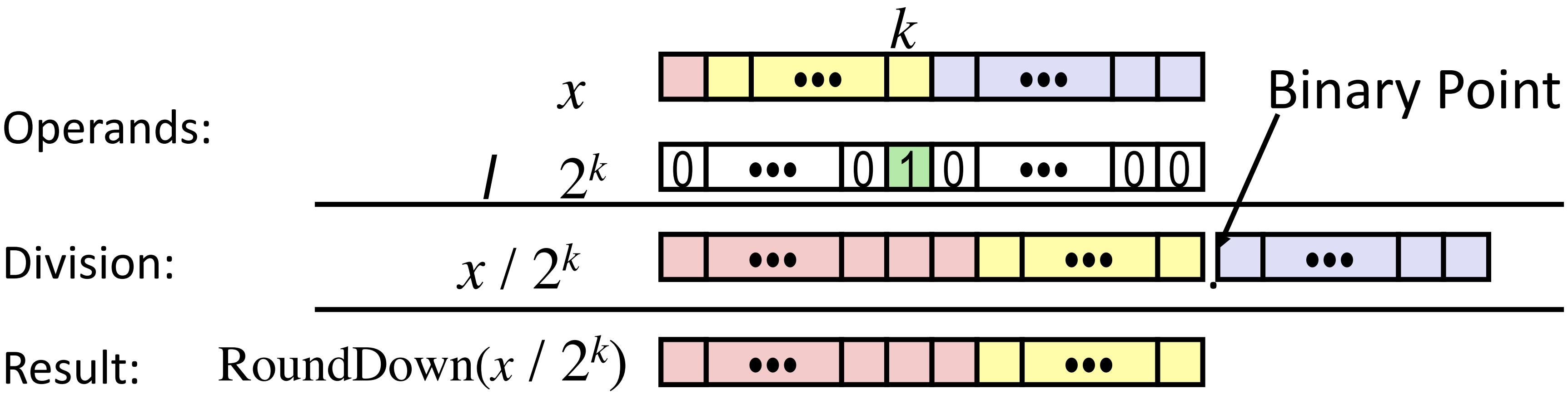
Explanation

```
  # Logical shift
  return x >> 3;
```

- **Uses logical shift for unsigned**
- **For Java Users**
  - Logical shift written as >>>

Signed Power-of-2 Divide with Shift

- **Quotient of Signed by Power of 2**
  - `x >> k` gives $\lfloor x / 2^k \rfloor$
  - Uses arithmetic shift *(in most cases – but the C standard does not demand this, see previous lecture!)*
  - Rounds wrong direction when `u < 0`

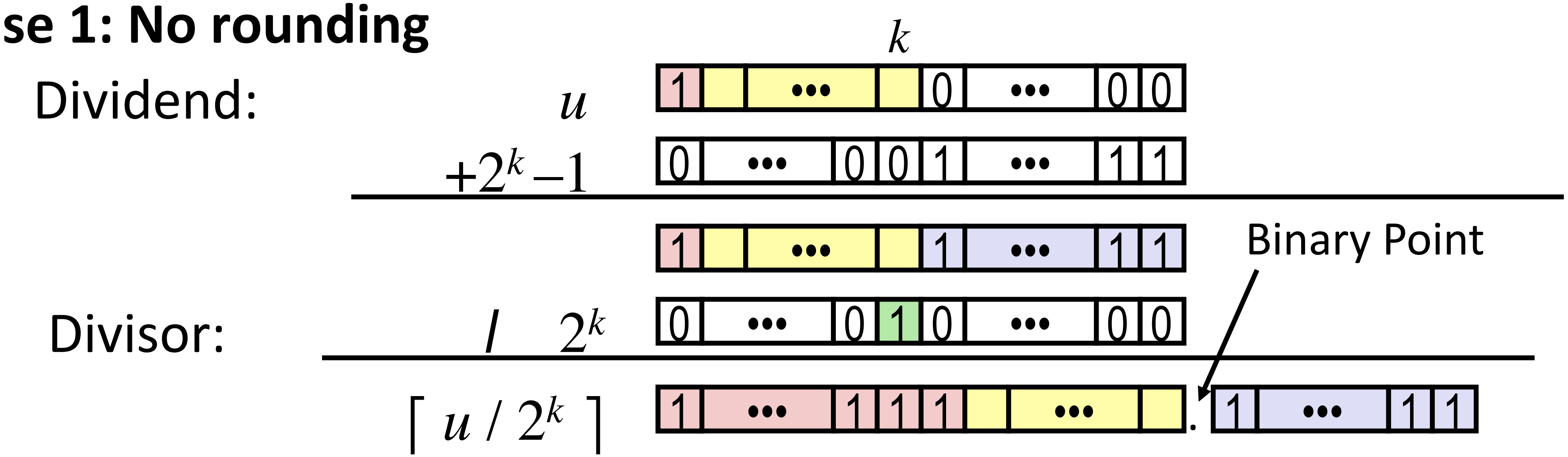


	Division	Computed	Hex	Binary
`y`	-15213	-15213	`C4 93`	`11000100 10010011`
`y >> 1`	-7606.5	-7607	`E2 49`	`11100010 01001001`
`y >> 4`	-950.8125	-951	`FC 49`	`11111100 01001001`
`y >> 8`	-59.4257813	-60	`FF C4`	`11111111 11000100`

Correct Power-of-2 Divide

- **Quotient of Negative Number by Power of 2**
  - Want $\lceil$ `x / 2`^{k} $\rceil$ (Round Toward 0)
  - Compute as $\lfloor$ `(x+2`^{k}`-1)/ 2`^{k} $\rfloor$
    - In C: `(x + (1<<k)-1) >> k`
    - Biases dividend toward 0

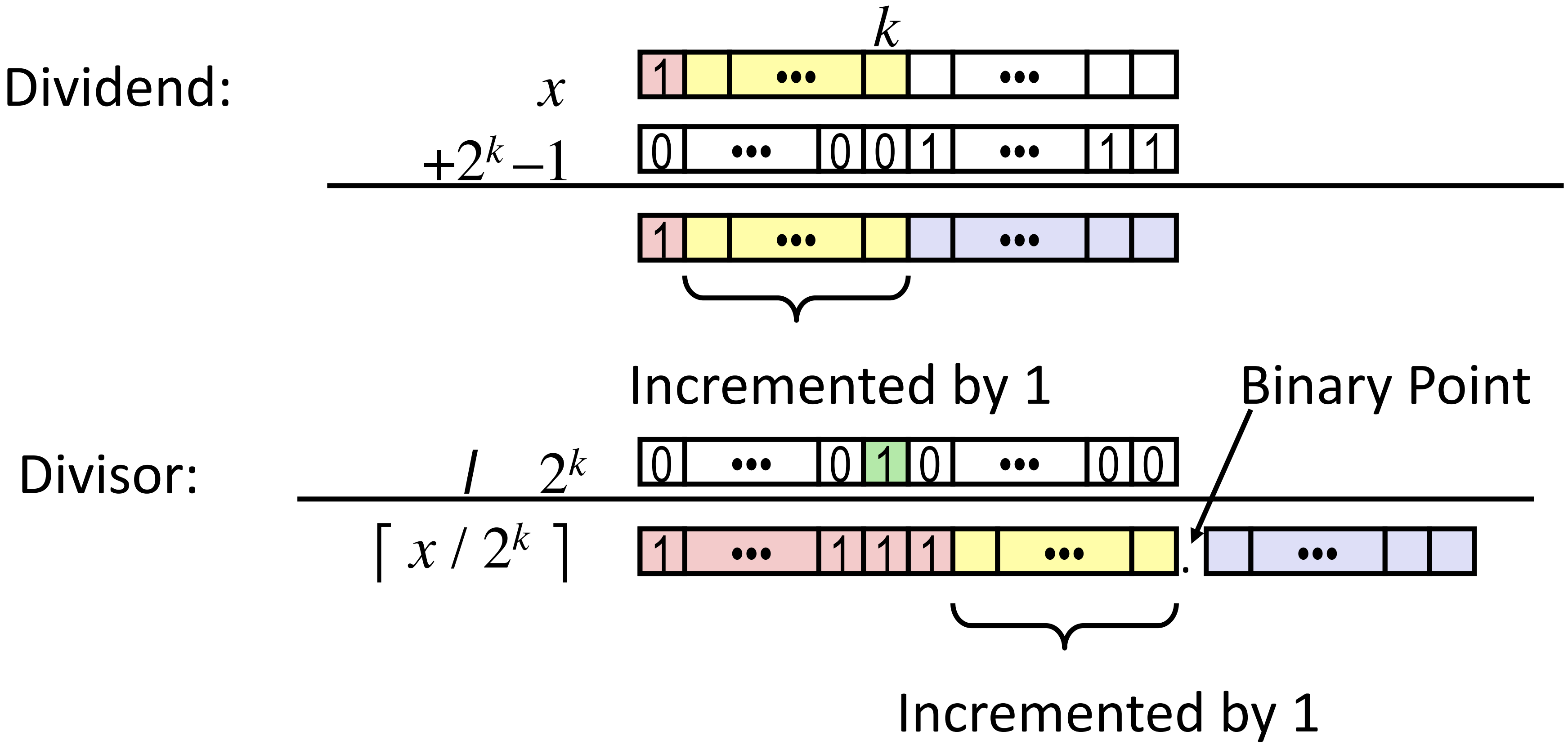
Case 1: No rounding



Biasing has no effect

Correct Power-of-2 Divide (Cont.)

Case 2: Rounding



Biasing adds 1 to final result

Compiled Signed Division Code

C Function

```
int idiv8(int x)
{
  return x/8;
}
```

Compiled Arithmetic Operations

```
  testl %eax, %eax
  js    L4
L3:
  sarl $3, %eax
  ret
L4:
  addl $7, %eax
  jmp   L3
```

Explanation

```
  if x < 0
    x += 7;
  # Arithmetic shift
  return x >> 3;
```

- **Uses arithmetic shift for int**
- **For Java Users**
  - Arith. shift written as >>

Arithmetic: Basic Rules

- **Addition:**
  - Unsigned/signed: Normal addition followed by truncate, same operation on bit level
  - Unsigned: addition mod 2^w
    - Mathematical addition + possible subtraction of 2^w
  - Signed: modified addition mod 2^w (result in proper range)
    - Mathematical addition + possible addition or subtraction of 2^w

- **Multiplication:**
  - Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
  - Unsigned: multiplication mod 2^w
  - Signed: modified multiplication mod 2^w (result in proper range)

Arithmetic: Basic Rules

- **Unsigned ints, 2's complement ints are isomorphic rings: isomorphism = casting**

- **Left shift**
  - Unsigned/signed: multiplication by 2^k
  - Always logical shift

- **Right shift**
  - Unsigned: logical shift, div (division + round to zero) by 2^k
  - Signed: arithmetic shift (*usually – the C standard does not demand this!*)
    - Positive numbers: div (division + round to zero) by 2^k
    - Negative numbers: div (division + round away from zero) by 2^k
      Use biasing to fix

Today: Integers

- Representation: unsigned and signed
- Conversion, casting
- Expanding, truncating
- Addition, negation, multiplication, shifting
- **Summary**

Properties of Unsigned Arithmetic

- **Unsigned Multiplication with Addition Forms Commutative Ring**
  - Addition is commutative group
  - Closed under multiplication

    $0 \leq \text{UMult}_w(u, v) \leq 2^w - 1$
  - Multiplication commutative

    $\text{UMult}_w(u, v) = \text{UMult}_w(v, u)$
  - Multiplication is associative

    $\text{UMult}_w(t, \text{UMult}_w(u, v)) = \text{UMult}_w(\text{UMult}_w(t, u), v)$
  - 1 is multiplicative identity

    $\text{UMult}_w(u, 1) = u$
  - Multiplication distributes over addition

    $\text{UMult}_w(t, \text{UAdd}_w(u, v)) = \text{UAdd}_w(\text{UMult}_w(t, u), \text{UMult}_w(t, v))$

Properties of Two's Comp. Arithmetic

- **Isomorphic Algebras**
  - Unsigned multiplication and addition
    - Truncating to *w* bits
  - Two's complement multiplication and addition
    - Truncating to *w* bits
- **Both Form Rings**
  - Isomorphic to ring of integers mod 2^w
- **Comparison to (Mathematical) Integer Arithmetic**
  - Both are rings
  - Integers obey ordering properties, e.g.,

$$u > 0 \quad \Rightarrow \quad u + v > v$$

$$u > 0, v > 0 \quad \Rightarrow \quad u \cdot v > 0$$

  - These properties are not obeyed by two's comp. arithmetic

TMax `+ 1` `==` *TMin*

`15213 * 30426 == -10030` (16-bit words)

Why Should I Use Unsigned?

- ***Don't*** **Use Just Because Number Nonnegative**
  - Easy to make mistakes

```
unsigned i;
for (i = cnt-2; i >= 0; i--)
  a[i] += a[i+1];
```

  - Can be very subtle

```
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
  . . .
```

- ***Do*** **Use When Performing Modular Arithmetic**
  - Multiprecision arithmetic
- ***Do*** **Use When Using Bits to Represent Sets**
  - Logical right shift, no sign extension

Signed/Unsigned Conventions in C

- **Constants: by default considered to be *signed integers***
- **If mix unsigned and signed in single expression,**
  signed values implicitly cast to unsigned

C Puzzle Answers

- Assume machine with 32 bit word size, two's comp. integers
- *TMin* makes a good counterexample in many cases

Initialization

```
int x = foo();
int y = bar();
unsigned ux = x;
unsigned uy = y;
```

❑ **x < 0**	⇒	**((x*2) < 0)**	**False:** *TMin*
❑ **ux >= 0**			**True:** 0 = *UMin*
❑ **x & 7 == 7**	⇒	**(x<<30) < 0**	**True:** $x_1 = 1$
❑ **ux > -1**			**False:** 0
❑ **x > y**	⇒	**-x < -y**	**False:** -1, *TMin*
❑ **x * x >= 0**			**False:** 30426
❑ **x > 0 && y > 0**	⇒	**x + y > 0**	**False:** *TMax*, *TMax*
❑ **x >= 0**	⇒	**-x <= 0**	**True:** $-TMax < 0$
❑ **x <= 0**	⇒	**-x >= 0**	**False:** *TMin*